

Calculus II Transformation Summary

Dr. Paul L. Bailey

September 4, 2017

Given a subset of \mathbb{R}^2 , we may modify the position or shape of the set by transforming each of the coordinates in each point of the set, according to the chart below.

Shift right h units	$x \mapsto x + h$
Shift left h units	$x \mapsto x - h$
Shift up k units	$y \mapsto y + k$
Shift down k units	$y \mapsto y - k$
Stretch horizontally by a factor of a	$x \mapsto ax$
Shrink horizontally by a factor of a	$x \mapsto \frac{x}{a}$
Stretch vertically by a factor of b	$y \mapsto by$
Shrink vertically by a factor of b	$y \mapsto \frac{y}{b}$
Reflect across the y -axis	$x \mapsto -x$
Reflect across the x -axis	$y \mapsto -y$

Transformations of Sets

Given an equation, we may transform the locus of the equation by modifying the original equations according to the rules laid out in the chart below.

Shift right h units	$x \mapsto x - h$
Shift left h units	$x \mapsto x + h$
Shift up k units	$y \mapsto y - k$
Shift down k units	$y \mapsto y + k$
Stretch horizontally by a factor of a	$x \mapsto \frac{x}{a}$
Shrink horizontally by a factor of a	$x \mapsto ax$
Stretch vertically by a factor of b	$y \mapsto \frac{y}{b}$
Shrink vertically by a factor of b	$y \mapsto by$
Reflect across the y -axis	$x \mapsto -x$
Reflect across the x -axis	$y \mapsto -y$

Transformations of Loci

Given a function f , we consider the equation $y = f(x)$ and obtain its locus. We may transform the graph of this locus by modifying the function according to the chart below.

Shift right h units	$f(x - h)$
Shift left h units	$f(x + h)$
Shift up k units	$f(x) + k$
Shift down k units	$f(x) - k$
Stretch horizontally by a factor of a	$f(\frac{x}{a})$
Shrink horizontally by a factor of a	$f(ax)$
Stretch vertically by a factor of b	$bf(x)$
Shrink vertically by a factor of b	$\frac{f(x)}{b}$
Reflect across the y -axis	$f(-x)$
Reflect across the x -axis	$-f(x)$

Transformations of Functions $f(x)$